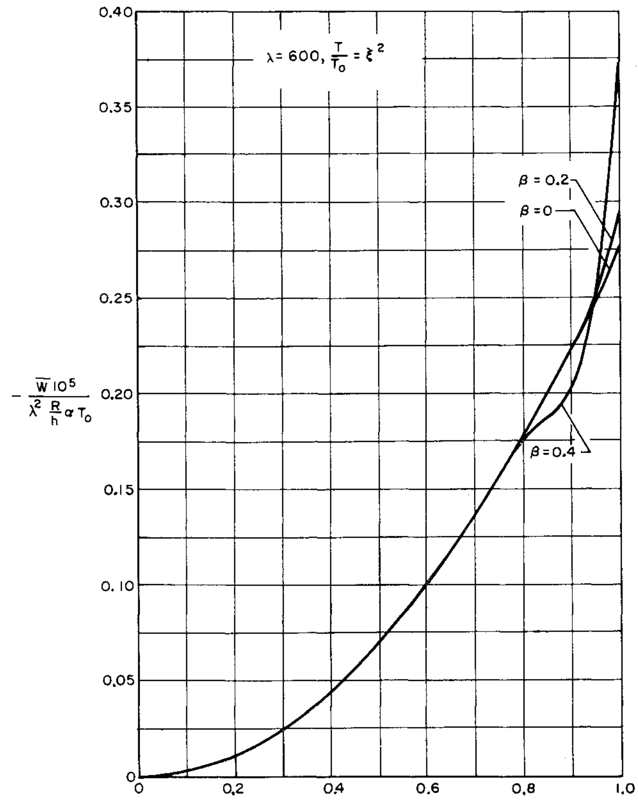

Fig. 1 Deflection vs axial position with end load as parameter.

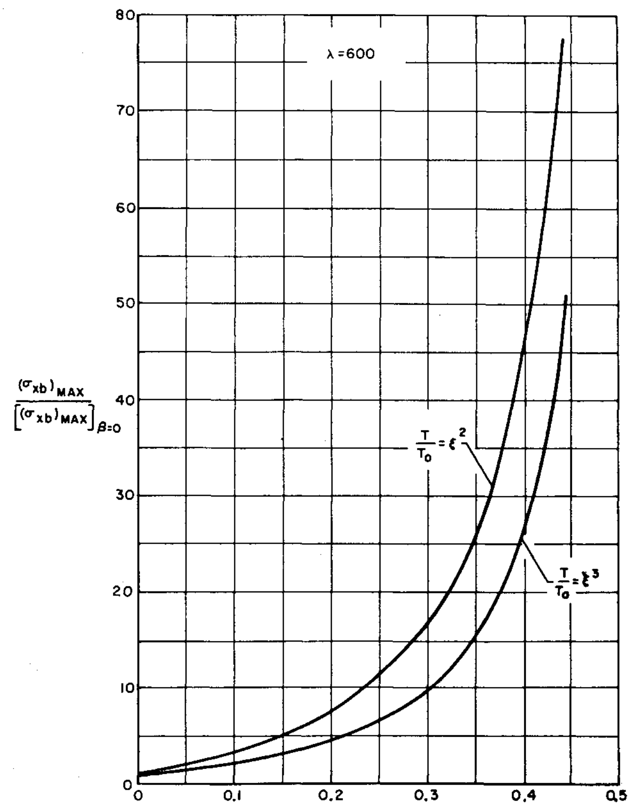

Fig. 2 Ratio of nondimensional bending stresses for parabolic and cubic temperature distributions.

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Radar Determination of Lunar Surface Dielectric Properties

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THEORETICAL reflection coefficient of an idealized spherical lunar surface is used to discuss ways and means to obtain its dielectric constants. Various estimates of these constants are compared, and then validity is checked. On the basis of this and other lunar theories, it is concluded that the presence or absence of a dust layer on the moon cannot be definitely established with available lunar echo data, although a fairly strong indication can be obtained by multifrequency experiments, from near the lunar surface, using lunar space probes.

The average distance of the moon from the earth is approximately 221 lunar radii, and therefore a spherical wave front can be safely approximated by a plane wave front at the surface of the moon. Moreover, it is well established[1,2] that nearly 50% of the lunar echo power is returned from the central area lying within a circle of radius 105 miles, or approximately $\frac{1}{10}$ of the radius of the moon. This leads to the approximation that the angle of incidence, measured from the outward surface normal to the propagation vector on the lunar surface is nearly zero. Assuming the surface permeability as unity, the Fresnel reflection factor R for a plane wave incident on a spherical surface is[6]

$$R = D_s^2 \left[\frac{(e_c)^{1/2} - 1}{(e_c)^{1/2} + 1}\right]^2 \tag{1}$$

where

D_s = divergence factor, in order to account for the spherical shape of the reflecting surface $\approx a/h$ (for vertical incidence)
a = radius of the sphere, (moon)
h = distance of the receiver from the moon surface
e_c = $(e/e_0) - j(s/we_0)$, relative complex dielectric constant of the central portion of the lunar surface
e_0 = free space dielectric constant
s = conductivity of lunar surface

The calculation of R/D_s^2 for a single frequency of transmission would result in an absolute value of e_c or

$$|e_c|^2 = (e/e_0)^2 + (s/we_0)^2 \tag{2}$$

It is possible to calculate the constants e and s for the lunar surface if a controlled lunar echo experiment is performed using transmission at several frequencies, which cover a range of about two octaves. The foregoing statements assume e and s to be approximately independent of frequency, in the absence of which such calculations become very complicated. It is pertinent to state that the dielectric properties so calculated would give only effective values for the

Received August 9, 1963.

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surface and would not indicate the presence or absence of surface layers. Therefore, it is not appropriate to classify the surface material on the basis of these effective dielectric constants.

Pettengill and Henry[7] assume the conductivity s to be zero and the permeability u to be unity and show that $e = 2.81$ for $(R/D_s^2) = 0.064$. Radio temperature measurements by Salomonovich[8] lead him to deduce the maximum value for e as 1.5. Senior and Siegel[9] use the plane wave power reflection factor R_1^2 from a dielectric slab under normal incidence conditions

$$R^2 = \left[\frac{1 - [(u_0/u)e_e]^{1/2}}{1 + [(u_0/u)e_e]^{1/2}}\right]^2 \tag{3}$$

and use a surface model made up of corner reflectors, cone-like projections, and flat portions to arrive at

$$e/u = 7.6 \times 10^{-6} \text{ mhos}^2$$

$$s/u = 2.7 \times 10^2 \text{ mhos/henry}$$

Numerous assumptions and "aesthetic" considerations used in their calculations reduce the value of their results to mere estimates although their approximated values, $e = 1.1\ e_0 = 9.6 \times 10^{-12}$ farads/m and $s = 3.4 \times 10^{-4}$ mhos/m for $u = u_0$, seem to fall in the range of other reported values.

The voltage reflection coefficient R for a sphere of radius a including an outer layer of thickness d is[10]

$$R = \frac{R_1 + R_2 \exp(-j2k_2d)}{1 + R_1R_2 \exp(-j2k_2d)} \tag{4}$$

where

R_1 = voltage reflection coefficient of the outer layer
R_2 = voltage reflection coefficient of the inner core
k_2 = $2\pi/L_2$, wave number
d = thickness of the outer layer on the sphere

and R_1 and R_2 are defined [from Eq. (3)] in terms of the dielectric constants as

$$R_1 = (1 - a)/(1 + a) \tag{5}$$

$$R_2 = (a - b)/(a + b) \tag{6}$$

where

$$a^2 = (u_0e_1/u_2e_0)[1 + j(s_1/w\ e_1)]$$

$$b^2 = (u_0e_2/u_2e_0)[1 + (s_1/w\ e_2)]$$

and where w is frequency in radians per second. These expressions can be used to solve for the six dielectric constants and the outer layer depth d from the previously suggested multifrequency experiment performed using a lunar orbiter type of satellite.

The depolarization of the electromagnetic waves incident at various angles on rough surfaces gives some indication of the type of roughness. In particular, linearly polarized incident signals become elliptically polarized by reflection from an absorbing medium, and this property may be used to determine the absorption coefficient of the lunar surface. Then the dielectric properties, the depth of the top surface layer, and the absorption coefficient could be correlated to determine the porosity and other physical properties of the surface material.

Pettengill and Henry[7] postulate that a relative permittivity of 2.81, calculated for their radar data taken at 68 cm wavelength, is similar to that of dry sand. Later experimental results (Evans and Pettengill[2]) seem to support this. Small-scale roughness indicated by $s/L = 0.1$ and $B/L = 1$ obtained (Hayre[3-5]) from Pettengill's results also seems to suggest that the lunar surface may have deformities that on the average may be approximately 68 cm in length and up to about 20 cm in height. It must be concluded that there is no unique method of definitely determining whether there is a dust layer or a sandy surface layer on the moon from either monostatic or bistatic radar studies or from albedo, temperature, and photometric function measurements. Nevertheless, such results as approximate depth of the top layer and the dielectric constants of the surface material may yield sufficient information to verify the design criterion for the Surveyor landing.

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Aerodynamic Coefficients in the Slip and Transition Regime

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Nomenclature

C_A = axial force coefficient
C_D = drag coefficient
C_M = pitching moment coefficient
D = base diameter
d = nose diameter
K_n = Knudsen number
M = Mach number, freestream
P = probability of no intermolecular collisions
Re = Reynolds number, freestream
U = velocity, freestream
X = aerodynamic coefficient
α = angle of attack
β = Martino number
δ^* = boundary-layer displacement thickness
Δ = shock detachment thickness
λ = mean free path
ρ = density

Subscripts

con = continuum
fm = free molecule
s = stagnation value
1 = freestream value

THE calculation of aerodynamic coefficients in the slip and transition regime is largely an inexact science. Some

Received August 6, 1963.

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